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Using the Shapley value of stocks as systematic risk

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Abstract

Purpose – This study aims to propose the Shapley value that originates from the game theory to quantify the relative risk of a security in an optimal portfolio.

Design/methodology/approach – Systematic risk as expressed by the relative covariance of stock returns to market returns is an essential measure in pricing risky securities. Although very much in use, the concept has become marginalized in recent years because of the difficulties that arise estimating beta. The idea is that portfolios can be viewed as cooperative games played by assets aiming at minimizing risk. With the Shapley value, investors can calculate the exact contribution of each risky asset to the joint payoff. For a portfolio of three stocks, this study exemplifies the Shapley value when risk is minimized regardless of portfolio return.

Findings – This study computes the Shapley value of stocks and indices for optimal mean-variance portfolios by using daily returns for the years 2016–2019. This results in the risk attributes allocated to securities in optimal portfolios. The Shapley values are analyzed and compared to the standard beta estimates to determine the ranking of assets with respect to pertinent risk and return.

Research limitations/implications – An alternative approach to value risk and return in optimal portfolios is presented in this study. The logic and the mechanics of Shapley value theory in portfolio analysis have been explained, and its advantages relative to standard beta analysis are presented. Hence, financial analysts when adding or removing specific assets from present positions will have the true and exact impact of their actions by using the Shapley value instead of the beta.

Practical implications – When computing the Shapley value, portfolio risk is decomposed exactly among its assets because it considers all possible coalitions of portfolios. In that sense, financial analysts when adding or removing specific securities from present holdings will be able to predict the true and exact impact of their transactions by using the Shapley value instead of the beta. The main implication for investors is that risk is ultimately priced relative to their holdings. This prevents the subjective mispricing of securities, as standard beta is not used and might allow investors to gain from arbitrage conditions.

Originality/value – The logic and the methodology of Shapley value theory in portfolio analysis have been explained as an alternative to value risk and return in optimal portfolios by presenting its advantages relative to standard beta analysis. The conclusion is that the Shapley value theory contributes much more financial optimization than to standard systematic risk analysis because it enables looking at the contribution of each security to all possible coalitions of portfolios.

Keywords Beta, Mean-variance frontier, Optimal portfolios

Paper type Research paper

1. Introduction

The purpose of this paper is to calculate the Shapley value (Shapley, 1953) of securities in optimal portfolios to estimate the systematic risk of individual stocks. The Shapley value emerges from cooperative game theory where it is used to measure the contribution of each player to the common goal of a game. Lloyd Shapley received the Nobel Prize in Economics for this result, and since then, the concept has been a standard theoretical and practical measure in economics, politics and sports as reviewed by Alvin Roth (1988). In finance, the Shapley value has been shown to be highly adequate for allocating costs of insurance

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companies (Lemaire, 1984), valuing corporate voting rights (Zingales, 1995; Nenova, 2003) and measuring the attribution of risk in banking systems (Tarashev *et al.*, 2015), to cite a few. However, applying the Shapley value in investments and portfolio theory has been quite limited. Only recently, Ortmann (2016) and Colini-Baleschi *et al.* (2018) used the Shapley theory to price the market risk of individual assets, and Shalit (2017) established the theoretical foundations for the Shapley value to decompose the risk of efficient portfolios. Shapley value does not change the way portfolios are optimized but rather computes the exact and true price to be attributed to individual assets. In this sense, the approach can be validated for use in classical mean-variance (MV) portfolios as presented here and can be used for any other investment model in the future.

My claim is that conventional betas measuring the sensitivity of stocks returns with respect to market returns are not sufficient to evaluate the pertinent risk of an asset in a well-diversified portfolio. Therefore, the subjective valuations that investors use to acquire these securities can be biased and lead to erroneous decisions. It is my contention that financial analysts would benefit a great deal by using Shapley values, as they could profit from the arbitrage conditions caused by price imbalances. Contrarily to standard beta analysis that considers the complete stock market, Shapley value imputes the contribution of each risky asset by looking for a given set of securities at all possible portfolios the security might participate in.

Indeed, by looking at portfolio selection as a cooperative game played by the securities for minimizing risk for a given return, one can compute the true and exact imputation of assets to the optimal portfolio. As risk reduction and return increase depend on the order stocks that are added to the portfolio, the Shapley value is calculated by averaging the marginal contributions from the arrival of stocks to the specific portfolios.

In the present paper, I construct efficient portfolios and extract the Shapley value of stocks in these optimal portfolios. The Shapley values are then compared to standard beta measures. I begin by presenting the mechanics of computing the Shapley value of risky assets, using a simple theoretical example first for minimum variance portfolios (MVPs), then for MV efficient portfolios. Indeed, before using the Shapley value in diversified investments, practitioners and financial analysts would benefit by understanding the working intricacies of the approach in risk reduction. Then, using daily returns of US stocks and financial indices for the years 2016-19, I construct optimal portfolios, estimate beta systematic risks and compare these figures to their Shapley values to evaluate their usefulness.

2. Shapley value in optimal portfolios

In this section, I present the methodology for evaluating the Shapley value of securities in a portfolio. Our investment model views portfolio selection as a cooperative game played by the securities to reach the common goal of minimizing portfolio risk for a given expected return or, alternatively, maximizing the portfolio's expected return for a specific risk. This is commonly known to be the standard MV portfolio optimization. What is less common is to consider portfolio selection as a cooperative game and then to use the Shapley value procedure to impute the value that each stock contributes to reduce portfolio risk. Shapley value considers all the portfolios a security can participate from itself alone to the total number of available stocks. We then calculate the stock's marginal contribution to risk reduction when it is added to a portfolio and, last, we average these contributions. The sum of these averages is the Shapley value for a given stock which is formulated below.

Let N be the total number of available stocks, s the number of stocks in a portfolio for s = 1, . . ., N, and $v(s)$ the risk associated with the optimal portfolio of s stocks. As securities are added to the portfolio, total risk decreases (at least not increases) as the result of diversification. For each stock, marginal risk differences are tabulated and, then, averaged using combinatorial probabilities since stocks can participate in portfolios in multiple ways. The averages are totaled to produce the Shapley value (SV) as follows for stock k:

$$SV_k(N,v) = \sum_{s=1}^{N-1} \sum_{S} \frac{s!(N-s-1)!}{N!} \left[v(S+k) - v(S)\right]) \tag{1}$$

Naturally, the sum of all the Shapley values of the assets equals the total risk of the portfolio built with these securities, which follows:

$$v(S) = \sum_{k=0}^{n} SV_k(N,v). \tag{2}$$

In the next example, I address several concerns a reader might be perplexed regarding the Shapley value formulation.

2.1 Shapley value in three stocks portfolio example

To understand the logic behind the Shapley value in investment analysis, we construct a portfolio of three hypothetical stocks A, B, C whose statistics are reported in Table 1. I will now demonstrate how the sum of incremental risks generated by a stock when added to the portfolio produces the Shapley value for that stock. For ease of presentation, consider as an investment objective the minimization of portfolio risk regardless of its mean return. This allocation results in the MVP that is derived in the following.

Regardless of portfolio mean returns μ_p, the objective is to choose the weights w_i to minimize the portfolio variance σ_p^2:

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \rho_{ij} \sigma_i \sigma_j \tag{3}$$

subject to the constraint that all the wealth is used up, i.e. $\sum_{i=1}^{N} w_i = 1$, allowing for short sales, $w \lesseqgtr 0$, where w_i is the weight of stock i, σ_i its standard deviation and ρ_{ij} is the coefficient of correlation between stock i and stock j. Using the matrix notation allows us to simplify the exposition (Fabozzi *et al.*, 2011). Then, the problem is stated as: minimize

Stocks	A	B	C
Correlation A	1	0	−0.3
Correlation B	0	1	0.4
Correlation C	−0.3	0.4	1
Mean (%)	10	5	20
Std Deviation (%)	30	10	20

Table 1. Example with three stocks

$\sigma_p^2 = \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}$ subject to $\boldsymbol{w}'\ \boldsymbol{l} = 1$ regardless of the portfolio mean return $\mu_p = \boldsymbol{w}'\ \boldsymbol{\mu}$, where $\boldsymbol{w}$ is the array of portfolio weights, $\boldsymbol{\Sigma}$ the variance-covariance matrix, $\boldsymbol{l}$ an array of ones and $\boldsymbol{\mu}$ the array of mean returns.

Using matrix notation, the optimal solution for MVP is explicitly determined in portfolio theory and practice (Huang and Litzenberger, 1988). With the quadratic forms $a = \boldsymbol{l}'\ \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$ and $c = \mathbf{1}'\ \boldsymbol{\Sigma}^{-1}\mathbf{1}$ the MVP solution yields the variance $\sigma^2(MVP) = 1/c$ and the mean $\mu(MVP) = a/c$. The MVP solution produces the portfolio weights 11.83% for Stock A, 76.72% for Stock B and 11.45% for Stock C. The standard deviation at MVP is given as 9.27% and its mean return 7.31%. To calculate the Shapley value of a stock at MVP, one needs to look at all the possible portfolios which contain the stock. We can see that for a set of three stocks, there are eight possible portfolios including the empty one.

We begin with the Shapley value of Stock A that is calculated by averaging the risks Stock A accrues in portfolios A, AB, AC and ABC. First, we add Stock A to an empty portfolio. The incremental risk is only its standard deviation σ_A, i.e., 30%. Then, we calculate the risk of portfolio AB at its MVP. The incremental risk of Stock A is calculated by adding A to B to form portfolio AB; that is, $\sigma_{AB}^{MVP} - \sigma_B = 9.49\% - 10\% = -0.51\%$. We repeat this procedure when A is added to C to form portfolio AC. Finally, we calculate the incremental risk of Stock A being added to portfolio BC to form ABC that is, $\sigma_{ABC}^{MVP} - \sigma_{BC}^{MVP} = 9.27\% - 9.94\% = -0.67\%$. The increments are averaged by taking into account the permutation probabilities. Hence, for Stock A, we obtain (1/3) 30% − (1/6) 0.51% − (1/6) 5.95% −(1/3) 0.67% = 8.69% The incremental risks and Shapley values for the three stocks in a minimum variance portfolio are then calculated and presented in Table 2.

As seen in Table 2, the sum of the Shapley values for the three stocks equals the standard deviation at MVP. We now compute the Shapley value for optimal MV portfolios. For that purpose, we need to calculate the efficient frontier, as done in the following.

2.2 Mean-variance optimal portfolios

Instead of minimizing the portfolio variance by itself, we construct optimal portfolios for specific mean returns. For simplicity of exposition, the optimization problem is expressed in matrix notation. The objective is to find the weights $\boldsymbol{w}$ that minimize half the portfolio variance $\frac{1}{2}\sigma_p^2 = \frac{1}{2}\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}$, subject to its mean return constraint $\mu_p = \boldsymbol{w}'\ \boldsymbol{\mu}$ and the portfolio constraint $\boldsymbol{w}'\ \boldsymbol{l} = 1$, allowing for short sales $\mathbf{w} \lesseqgtr 0$.

As shown by Pachamanova and Fabozzi (2016) to cite a recent reference, the optimization solution is a set of portfolio weights that depend on the required mean return μ_p as follows:

$$\boldsymbol{w}_p^* = \frac{1}{d}\left[b \cdot \boldsymbol{\Sigma}^{-1}\boldsymbol{l} - a \cdot \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}\right] + \left[c\boldsymbol{\Sigma}^{-1}\boldsymbol{l} - a \cdot \boldsymbol{\Sigma}^{-1}\boldsymbol{l}\right]\mu_p. \quad (4)$$

Table 2.
MVP and Shapley values quoted in standard deviation

Portfolio	Probability	MVP (%)	Incremental A (%)	Incremental B (%)	Incremental C (%)
A	1/3	30	+30		
B	1/3	10		+10	
C	1/3	20			+20
AB	1/6	9.49	−0.51	−20.51	
AC	1/6	14.05	−5.95		−15.95
BC	1/6	9.94		−10.06	−0.06
ABC	1/3	9.27	−0.67	−4.78	−0.22
Shapley value		9.27	8.69	−3.36	3.92

where the quadratic forms a and c are as shown above, $b = \boldsymbol{\mu}' \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}$ and $d = bc - a^2$.

Based on the above, one can delineate the MV efficient frontier by expressing portfolio risk σ_p for a given return μ_p as:

$$\sigma_p = \sqrt{\frac{c}{d}\left(\mu_p - \frac{a}{c}\right)^2 + \frac{1}{c}}. \tag{5}$$

Using the data in Table 2, the frontier of MV optimal portfolios is calculated and is reported for a select number of required returns on Table 3. As one moves along the frontier from a relatively safe portfolio such as Portfolio II to a riskier one with higher mean return and higher variance such as Portfolio VI, the weight of Stock A somewhat increases, the weight of Stock C increases more and the holdings of Stock B are reduced and even shorted. This is because of the fact that Stock B is considered the safest asset and C the riskiest one. The frontier is depicted in Figure 1.

Now, one can decompose the risk of frontier portfolios by attributing the Shapley value for each of the three stocks to the standard deviation of the optimal portfolios. This is done by using equation (1). The results are shown in Table 4 for the selected optimal portfolios along the frontier.

According to the results of Table 4, we can see that, as expected from MV portfolio theory, the higher the required expected return is the higher is the risk exhibited by optimal portfolios. What is less obvious is the relative contribution of the various stocks to portfolio risk. Some Shapley values are negative meaning that some stocks reduce portfolio risk as mean return increases, while other values are positive as their stocks contribute to increasing risk. What is remarkable is that the Shapley value reveals that the asset's relative contribution to portfolio risk evolves along the optimal frontier. Table 5 exhibits the Shapley

Portfolio	Mean (%)	SD (%)	Stock A (%)	Stock B (%)	Stock C (%)
I	7.31	9.27	11.83	76.72	11.45
II	10.31	9.84	16.61	53.53	29.86
III	13.31	11.39	21.39	30.35	48.27
IV	16.31	13.59	26.16	7.16	66.67
V	19.31	16.18	30.94	−16.02	85.08
VI	22.31	18.99	35.72	−39.20	103.49

Table 3. Weights of Selected Frontier Portfolios

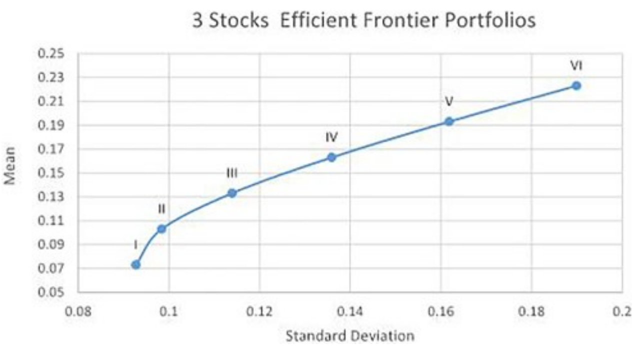

Figure 1. 3 Stocks efficient frontier portfolios

values relative to the portfolio standard deviation. For Stock A, the relative contribution stays somehow constant whereas the relative contribution of Stock B increases along the frontier while the relative contribution of Stock C decreases.

3. Systematic risk in actual portfolios

Before calculating the Shapley value for stocks in actual portfolios, I recapitulate the essence of systematic risk in investments. In portfolio theory, the concept used for systematic risk is a measure of asset riskiness relative to market performance since securities are held in portfolios. This is the risk most relevant for evaluating and pricing risky assets. The typical measure is the beta risk obtained by the relative covariation of asset returns and market portfolio returns, namely, $\beta_i = \text{cov}(r_i, r_m)/\sigma_m^2$, where r_i are the stock returns, r_m the market returns and σ_m^2 the variance of market returns. The underlying assumption is that the entire stock market is used as the investor's portfolio of assets. The alternative approach I propose here would be to express systematic risk as the relative risk a security contributes to a specific portfolio for a given number of assets as expressed by the Shapley value.

The analysis is conducted on a portfolio of 13 stocks and industry indices such as exchange traded funds (ETFs) that encompass most of the US stock market. The collected data consist of daily adjusted returns (accounting for dividends and other distributions) for the years 2016, 2017, 2018 and 2019 whose statistics are shown on Table 6. I calculate the securities' systematic risk using two approaches. The first is the standard method where market returns are proxied by the returns on a market index such as the S&P500. For that purpose, I use the returns on the S&P500 ETF whose symbol is SPY. The results are compiled in Table 6.

Instead of market returns, the second approach is to use the returns on the optimal MV portfolio for a given mean. To do so, one optimizes the portfolio for a given mean return. The portfolio returns are computed using the optimal portfolio weights from equation (4) as computed in Table 7 for six arbitrary means. Once the returns are obtained they are used to calculate the relative covariation of the stock with respect to the optimal portfolio to obtain the betas for the individual stocks. The results are presented in Table 6 for the portfolio P* whose mean return is 0.052% [1].

Table 4. Shapley Values of stocks on the frontier portfolios

Portfolio	Mean (%)	SD (%)	Shapley A (%)	Shapley B (%)	Shapley C (%)
I	7.31	9.27	13.87	−11.09	6.49
II	10.31	9.84	14.63	−4.15	−0.64
III	13.31	11.39	15.87	3.04	−7.52
IV	16.31	13.59	17.97	9.05	−13.43
V	19.31	16.18	21.63	12.21	−17.66
VI	22.31	18.99	26.20	13.70	−20.91

Table 5. Relative Shapley values

Portfolio	Mean (%)	SD (%)	% Shapley A	% Shapley B	% Shapley C
I	7.31	9.27	149.62	−119.63	70.01
II	10.31	9.84	148.68	−42.17	−6.50
III	13.31	11.39	139.33	26.69	−66.02
IV	16.31	13.59	132.23	66.59	−98.82
V	19.31	16.18	133.68	75.46	−109.15
VI	22.31	18.99	137.97	72.14	−110.11

Symbol	Name	Mean (%)	SD (%)	β w.r.t. SPY	β w.r.t. port II*
AAPL	Apple Inc	0.12	1.53	1.27	2.02
BAC	Bank of America	0.09	1.62	1.36	1.63
EEM	Emerging Market ETF	0.05	1.14	1.06	0.94
FB	Facebook Inc	0.08	1.82	1.22	1.47
IWM	Russell 2000 ETF	0.05	1.02	1.09	0.95
JNK	Hi Yield Bond ETF	0.03	0.36	0.32	0.68
QQQ	NASDAQ-100 ETF	0.07	1.07	1.22	1.32
XLE	Energy ETF	0.02	1.28	1.08	0.55
XLF	Financial ETF	0.09	1.44	1.09	1.56
XLK	Technology ETF	0.09	1.10	1.24	1.54
XLU	Utilities ETF	0.06	0.83	0.26	1.06
XLV	Health Care ETF	0.05	0.89	0.90	0.90
XOP	Exploration Oil ETF	0.00	2.14	1.52	0.23

Table 6.
Systematic risks of selected assets 2016-2019

Portfolios	I (%)	II (%)	III (%)	IV (%)	V (%)	VI (%)
Mean return	0.0282	0.0520	0.0757	0.0995	0.1233	0.1471
SD	0.2996	0.3740	0.5387	0.7354	0.9442	1.1586
AAPL	1.40	8.48	15.56	22.63	29.71	36.79
BAC	−2.21	6.35	14.90	23.45	32.00	40.55
EEM	−9.55	−11.94	−14.34	−16.73	−19.13	−21.52
FB	0.78	2.96	5.15	7.34	9.52	11.71
IWM	−1.13	−7.72	14.30	−20.88	−27.47	−34.05
JNK	114.94 9	5.63	76.33	57.03	37.73	18.42
QQQ	−11.05 −	42.03	−73.02	−104.01	−135.00	−165.98
XLE	1.48	−3.35	−8.18	−13.01	−17.84	−22.67
XLF	1.38	4.91	8.43	11.96	15.49	19.01
XLK	0.07	36.06	72.05	108.04	144.03	180.02
XLU	3.16	15.19	27.22	39.25	51.29	63.32
XLV	5.80	1.62	−2.56	−6.74	−10.91	−15.09
XOP	−5.08	−6.16	−7.25	−8.34	−9.43	−10.52

Table 7.
Assets holdings of six frontier portfolios

Because systematic risk is the value investors prefer to use when managing the riskiness of assets in portfolios, it is important it reflects its original intention. My contention is that beta as systematic risk might fail when ranking the relative riskiness of assets because it considers only the risk imputation in the final portfolio and not all possible portfolios the stock might be included in. Using the Shapley value to decompose portfolio risk into specific components should remedy these lacunae.

4. Shapley value in actual portfolios

The basic idea behind the Shapley value in portfolio analysis is to decompose the risk of optimal portfolios such that they are aptly attributed to individual stocks. The outcome of this decomposition reveals the relevant risk investors should use when managing portfolios. The conception that only risk matters regardless of mean return is not pertinent here because one analyzes MV efficient portfolios, i.e. they are portfolios that minimize the variance for given mean return. Using a data set of 13 selected stocks and ETFs, we calculate the MV portfolio frontier using the procedure outlined in Section 2.2. The assets weights for six data points on the efficient frontier are presented in Table 7.

To calculate the Shapley value of assets on the efficient frontier, we follow the optimization procedure outlined in Section 2.2 for all the *subsets* of the assets in the portfolio. Specifically, we first establish all the 2^N subsets of the securities in set N. Then, we compute the variance-covariance matrix $\boldsymbol{\Sigma}$, $a = \boldsymbol{l}' \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}$, $b = \boldsymbol{\mu}' \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}$, $c = \boldsymbol{l}' \boldsymbol{\Sigma}^{-1} \boldsymbol{l}$ and $d = bc - a^2$ for all the *subsets*. For some arbitrary required mean return μ_p like the ones shown on the first row of Table 7 we calculate for all the *subsets* the optimal standard deviation expressed by equation (5), i.e., $\sigma_p = \sqrt{\frac{c}{d}\left(\mu_p - \frac{a}{c}\right)^2 + \frac{1}{c}}$. This is the risk that is used to compute the Shapley values of equation (1).

The results for the 13 selected stocks and ETFs are shown on Table 8 for four frontier portfolios. We can see that some Shapley values are negative implying that those assets reduce their risk contribution to the portfolio. This is mostly the case for ETFs. Positive Shapley values imply that these assets increase risk along the optimal frontier and, therefore, increase mean return. To assess the usefulness of Shapley values in optimal portfolios we need to look at the assets contribution relative to portfolio risk. This feature is shown in Table 9 for four optimal portfolios on the efficient frontier. For our Portfolio II on the frontier, we can see that the assets contributing the most to risk and return during the 2016-2019 sample span were Facebook Inc, the Financial ETF and the Technology ETF. We then compare the results of the Shapley value approach with conventional systematic risk by ranking the assets with respect to beta and the Shapley value as exhibited in Table 10. For the 2016–2019 years, ranking the 13 assets differ somewhat since conventional beta relates to the risk and return of the entire market proxied by the SPY ETF whereas the relative Shapley value ranking is specific to the optimal portfolio constructed with the 13 assets, as it imputes relative risk and return between the others assets in the portfolio.

In portfolio theory, ranking according to beta explains how a specific asset reacts with respect to the entire stock market's movement. In contrast, pricing securities according to Shapley value theory reflects the optimal decomposition of portfolio risk. Within our data, ranking according to relative Shapley values show that the Financial ETF XLF, the Technology ETF XLK and Facebook Inc were the most valuable assets in increasing risk and return for that small efficient portfolio during the 2016-2019 period.

Table 8. Shapley values of frontier portfolios assets

Portfolios	I (%)	II (%)	III (%)	IV (%)
Mean return	0.0282	0.0520	0.0757	0.0995
SD	0.2996	0.3740	0.5387	0.7354
AAPL	−0.0654	0.0206	−0.0523	−0.2072
BAC	0.2817	0.2204	0.0305	−0.0876
EEM	0.0866	−0.0298	0.3461	0.6599
FB	0.4024	0.2880	0.1030	0.0758
IWM	0.0381	−0.0606	0.2976	0.5845
JNK	−0.7842	−0.3935	−0.1455	−0.0668
QQQ	−0.1026	−0.0509	−0.1007	−0.1479
XLE	−0.2170	−0.0565	0.0147	−0.0023
XLF	0.6310	0.4120	0.0771	−0.0202
XLK	0.4925	0.3214	−0.0113	−0.1562
XLU	−0.1661	−0.2062	−0.1327	−0.0552
XLV	−0.2049	−0.1137	0.0645	0.1600
XOP	−0.0925	0.0229	0.0478	−0.0015

Portfolios	I (%)	II (%)	III (%)	IV (%)
Mean return	0.0282	0.0520	0.0757	0.0995
SD	0.2996	0.3740	0.5387	0.7354
AAPL	−0.2182	0.0551	−0.0972	−0.2818
BAC	0.9402	0.5893	0.0566	−0.1191
EEM	0.2892	−0.0798	0.6424	0.8974
FB	1.3428	0.7700	0.1912	0.1031
IWM	0.1271	−0.1620	0.5524	0.7949
JNK	−2.6170	−1.0520	−0.2700	−0.0909
QQQ	−0.3425	−0.1362	−0.1869	−0.2011
XLE	−0.7242	−0.1511	0.0272	−0.0031
XLF	2.1057	1.1015	0.1431	−0.0274
XLK	1.6437	0.8593	−0.0210	−0.2124
XLU	−0.5543	−0.5513	−0.2463	−0.0751
XLV	−0.6839	−0.3039	0.1198	0.2176
XOP	−0.3087	0.0612	0.0886	−0.0021

Table 9. Relative shapley value (SV) of frontier portfolio assets

Assets ranked wrt to Shapley		Assets ranked wrt to Beta	
Symbol	Relative SV	Symbol	β (SPY)
JNK	−1.0520	XLU	0.26
XLU	−0.5513	JNK	0.32
XLV	−0.3039	XLV	0.90
IWM	−0.1620	EEM	1.06
XLE	−0.1511	XLE	1.08
QQQ	−0.1362	XLF	1.09
EEM	−0.0798	IWM	1.09
AAPL	0.0551	QQQ	1.22
XOP	0.0612	FB	1.22
BAC	0.5893	XLK	1.24
FB	0.7700	AAPL	1.27
XLK	0.8593	BAC	1.36
XLF	1.1015	XOP	1.52

Table 10. Relative Shapley values vs systematic risk

5. Some concluding remarks

I have explained the logic and the methodology of Shapley value theory in portfolio analysis as an alternative to value risk and return in optimal portfolios by presenting its advantages relative to standard beta analysis. My conclusion is that the Shapley value theory contributes more to financial optimization than to standard systematic risk analysis because it enables looking at the contribution of each security to all possible coalitions of portfolios.

Shapley value in investments stands at the culmination of an evolution that links asset returns to pertinent risk. Initially, investors first looked at the security own risk and variance to price expected return. Then, with Markowitz (1952) theory, portfolios began to be constructed using correlations and covariances. With the advent of capital asset pricing model, investors priced assets by comparing them to the entire stock market and computing systematic risk. When computing the Shapley value, portfolio risk is decomposed exactly among its assets because it considers all possible coalitions of portfolios. In that sense, financial analysts when adding or removing specific securities from present holdings will be able to predict the true and exact impact of their transactions by using the Shapley value instead of the beta. The main implication for investors is that risk is ultimately priced relative to their holdings. This prevents the subjective mispricing of securities, as standard beta is not used and might allow investors to gain from arbitrage conditions.

Note

1. As a reference, the mean return on the SPY index for the 2016-2019 years was 0.056%.

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Further reading

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